More Bits and Bytes
Huffman Coding

Encoding Text: How is it done?
ASCII, UTF, Huffman algorithm

ASCII

0100 0011
0100 0001
0101 0100

C
A
T

ASCII	0000	0001	0010	0011	0100	0101	0110	0111	1000	1001	1010	1011	1100	1101	1110	1111
0000	NU	SH	SX	EX	ET	EQ	AK	BL	BS	HT	LF	YT	FF	CR	S0	SI
0001	DL	D1	D2	D3	D4	NK	SY	EΣ	CN	EM	SB	EC	FS	GS	RS	US
0010		!	"	#	$	%	&	'	(	)	*	+	,	-	.	/
0011	0	1	2	3	4	5	6	7	8	9	:	;	<	=	>	?
0100	@	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O
0101	P	Q	R	S	T	U	V	W	X	Y	Z	[	\	]	^	_
0110	`	a	b	c	d	e	f	g	h	i	j	k	l	m	n	o
0111	p	q	r	s	t	u	v	w	x	y	z	{	|	}	~	DT
1000	80	81	82	83	IN	NL	SS	ES	HS	HJ	YS	PD	PV	RI	S2	S3
1001	DC	P1	PZ	SE	CC	MM	SP	EP	Q8	QQ	QA	CS	ST	OS	PM	AP
1010	AO	¡	¢	£	♀	¥	¦	§	¨	©	♂	«	¬	-	®	¯
1011	°	±	²	³	´	µ	¶	·	¸	¹	º	»	¼	½	¾	¿
1100	À	Á	Â	Ã	Ä	Å	Æ	Ç	È	É	Ê	Ë	Ì	Í	Î	Ï
1101	Ð	Ñ	Ò	Ó	Ô	Õ	Ö	×	Ø	Ù	Ú	Û	Ü	Ý	Þ	ß
1110	à	á	â	ã	ä	å	æ	ç	è	é	ê	ë	ì	í	î	ï
1111	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]

0100 0111 | 0110 1111 | 0010 0000 | 0101 0011 | 0110 1100 | 0111 0101 | 0110 0111 | 0111 0011 |

UTF-8: All the alphabets in the world

- Uniform Transformation Format: a variable-width encoding that can represent every character in the Unicode Character set
- 1,112,064 of them!!!
- http://en.wikipedia.org/wiki/UTF-8
- UTF-8 is the dominant character encoding for the World-Wide Web, accounting for more than half of all Web pages.
- The Internet Engineering Task Force (IETF) requires all Internet protocols to identify the encoding used for character data
- The supported character encodings must include UTF-8.

لماذا لا يتكلمون اللّغة العربية فحسب؟
Защо те просто не могат да говорят **български**?
Per què no poden simplement parlar en **català**?
他們爲什麼不說中文（**台灣**）？
Proč prostě nemluví **česky**?
Hvorfor kan de ikke bare tale **dansk**?
Warum sprechen sie nicht einfach **Deutsch**?
Μα γιατί δεν μπορούν να μιλήσουν **Ελληνικά**;
Why can't they just speak **English**?
¿Por qué no pueden simplemente hablar en **castellano**?
Miksi he eivät yksinkertaisesti puhu **suomea**?
Pourquoi, tout simplement, ne parlent-ils pas **français** ?
למה הם פשוט לא מדברים **עברית**?
Miért nem beszélnek egyszerűen **magyarul**?
Af hverju geta þeir ekki bara talað **íslensku**?
Perché non possono semplicemente parlare **italiano**?
なぜ、みんな**日本語**を話してくれないのか？
세계의 모든 사람들이 **한국어**를 이해한다면 얼마나 좋을까?
Waarom spreken ze niet gewoon **Nederlands**?
Hvorfor kan de ikke bare snakke **norsk**?
Dlaczego oni po prostu nie mówią po **polsku**?
Porque é que eles não falam em **Português (do Brasil)**?
Oare ăştia de ce nu vorbesc **româneşte**?
Почему же они не говорят **по-русски**?
Zašto jednostavno ne govore **hrvatski**?
Pse nuk duan të flasin vetëm **shqip**?
Varför pratar dom inte bara **svenska**?
ทำไมเขาถึงไม่พูด**ภาษาไทย**
Neden **Türkçe** konuşamıyorlar?

UTF is a VARIABLE LENGTH ALPHABET CODING

- Remember ASCII can only represent 128 characters (7 bits)
- UTF encodes over one million
- Why would you want a variable length coding scheme?

Bits	Last code point	Byte 1	Byte 2	Byte 3	Byte 4	Byte 5	Byte 6
7	U+007F	0xxxxxxx					
11	U+07FF	110xxxxx	10xxxxxx				
16	U+FFFF	1110xxxx	10xxxxxx	10xxxxxx			
21	U+1FFFFF	11110xxx	10xxxxxx	10xxxxxx	10xxxxxx		
26	U+3FFFFFF	111110xx	10xxxxxx	10xxxxxx	10xxxxxx	10xxxxxx	
31	U+7FFFFFFF	1111110x	10xxxxxx	10xxxxxx	10xxxxxx	10xxxxxx	10xxxxxx

UTF-8

Bits of code point	First code point	Last code point	Bytes in sequence	Byte 1	Byte 2	Byte 3	Byte 4
7	U+0000	U+007F	1	0xxxxxxx			
11	U+0080	U+07FF	2	110xxxxx	10xxxxxx		
16	U+0800	U+FFFF	3	1110xxxx	10xxxxxx	10xxxxxx	
21	U+10000	U+1FFFFF	4	11110xxx	10xxxxxx	10xxxxxx	10xxxxxx

UTF-8

Bits of code point	First code point	Last code point	Bytes in sequence	Byte 1	Byte 2	Byte 3	Byte 4
7	U+0000	U+007F	1	0xxxxxxx			
11	U+0080	U+07FF	2	110xxxxx	10xxxxxx		
16	U+0800	U+FFFF	3	1110xxxx	10xxxxxx	10xxxxxx	
21	U+10000	U+1FFFFF	4	11110xxx	10xxxxxx	10xxxxxx	10xxxxxx

What is the first Unicode value represented by this sequence?
11101010 10000011 10000111 00111111 11000011 10000000

A. 0000000001101010
B. 0000000011101010
C. 0000001010000111
D. 1010000011000111

UTF-8

Bits of code point	First code point	Last code point	Bytes in sequence	Byte 1	Byte 2	Byte 3	Byte 4
7	U+0000	U+007F	1	0xxxxxxx			
11	U+0080	U+07FF	2	110xxxxx	10xxxxxx		
16	U+0800	U+FFFF	3	1110xxxx	10xxxxxx	10xxxxxx	
21	U+10000	U+1FFFFF	4	11110xxx	10xxxxxx	10xxxxxx	10xxxxxx

How many Unicode characters are represented by this sequence?
11101010 10000011 10000111 00111111 11000011 10000000

A. 1
B. 2
C. 3
D. 4
E. 5

How many bits for all of Unicode?

There are 1,112,064 different Unicode characters. If a fixed bit format (like ascii with its 7 bits) where used, how many bits would you need for each character? (Hint: $2^{10} = 1024$)

A. 10
B. 17
C. 21
D. 32
E. 40

A Curious Story…

The Diving Bell and the Butterfly

Jean-Dominique Bauby

Asking Yes/No Questions

- A protocol for Yes/No questions
  - One blink == Yes
  - Two blinks == No
- PandA implies that this is not the fewest number of blinks … really?

Asking Letters

In English ETAOINSHRDLU…

Compare Two Orderings

- How many questions to encode:

  Plus ça change, plus c'est la même chose?

- Asking in Frequency Order:
  ESARINTULOMDPCFBVHGJQZYXKW

9 12

Compare Two Orderings

- How many questions to encode:

    Plus ça change, plus c'est la même chose?

- Asking in Frequency Order:
  ESARINTULOMDPCFBVHGJQZYXKW
- Asking in Alphabetical Order:

  ABCDEFGHIJKLMNOPQRSTUVWXYZ

  ↑ 12 (at L)　　↑ 16 (at P)

Compare Two Orderings

- How many questions to encode:

  Plus ça change, plus c'est la même chose?

- Asking in Frequency Order: 247
  ESARINTULOMDPCFBVHGJQZYXKW
- Asking in Alphabetical Order: 324

  ABCDEFGHIJKLMNOPQRSTUVWXYZ

An Algorithm

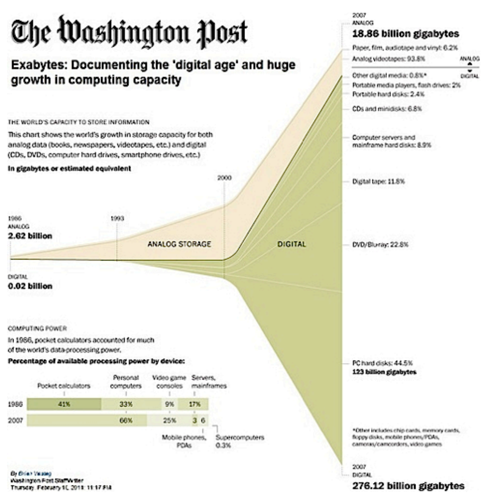
- Spelling by going through the letters is an algorithm
- Going through the letters in frequency order is a program (also, an algorithm but with the order specified to a particular case, i.e. FR)
- The nurses didn't look this up in a book … they invented it to make their work easier; they were thinking computationally, though they probably didn't know it

Coding can be used to do Compression

- What is CODING?
  - The conversion of one representation into another
- What is COMPRESSION?
  - Change the representation (digitization)

  in order to **reduce** size of data

  (number of bits needed to represent data)



- Benefits
  - **Reduce storage** needed
    - Consider growth of digitized data.
  - **Reduce** transmission cost / **latency /** bandwidth
  - When you have a 56K dialup modem, every savings in BITS counts, **SPEED**
    - Also consider telephone lines, texting

What makes it possible to do Compression?

- IN OTHER WORDS: When is Coding USEFUL?
- When there is **Redundancy**
  - Recognize repeating patterns
  - Exploit using
    - Dictionary
    - Variable length encoding
- When **Human perception is less sensitive** to some information
  - Can discard less important data

How easy is it to do it?

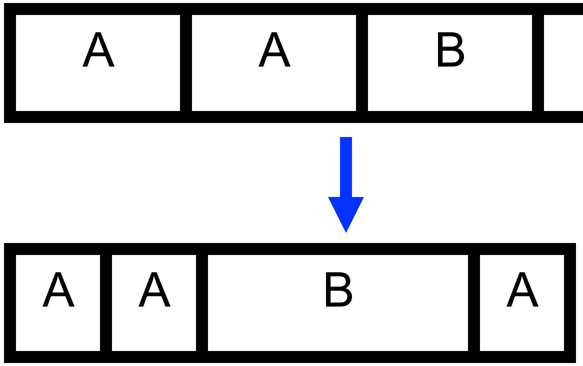
- Depends on data
  - Random data ⇒ hard
    - Example: 1001110100 ⇒ ?
  - Organized data ⇒ easy
    - Example: 1111111111 ⇒ 1×10
- WHAT DOES THAT MEAN?
  - **There is NO universally best compression algorithm**
  - **It depends on how tuned the coding is to the data you have**

Can you lose information with Compression?

- **Lossless** Compression is not guaranteed
  - Pigeonhole principle
    - Reduce size 1 bit ⇒ can only store ½ of data
    - Example
      - 000, 001, 010, 011, 100, 101, 110, 111 ⇒ 00, 01, 10, 11
  - CONSIDER THE ALTERNATIVE
  - IF LOSSLESS COMPRESSION WERE GUARANTEED THEN
    - Compress file (reduce size by 1 bit)
    - Recompress output
    - Repeat (until we can store data with 0 bits)
  - OBVIOUS CONTRADICTION => IT IS NOT GUARANTEED.

Huffman Code: A Lossless Compression

- Use Variable Length codes based on frequency (like UTF does)
- Approach
  - Exploit statistical frequency of symbols
  - What do I MEAN by that? WE COUNT!!!
- HELPS when the frequency for different symbols varies widely
- Principle
  - Use fewer bits to represent frequent symbols
  - Use more bits to represent infrequent symbols



Huffman Code Example

- "dog cat cat bird bird bird bird fish"

Symbol	Dog	Cat	Bird	Fish
Frequency	1/8	1/4	1/2	1/8
Original Encoding	00	01	10	11
	2 bits	2 bits	2 bits	2 bits
Huffman Encoding	110	10	0	111
	3 bits	2 bits	1 bit	3 bits

- Expected size
  - Original $\Rightarrow$ $1/8 \times 2 + 1/4 \times 2 + 1/2 \times 2 + 1/8 \times 2 = 2$ bits / symbol
  - Huffman $\Rightarrow$ $1/8 \times 3 + 1/4 \times 2 + 1/2 \times 1 + 1/8 \times 3 = 1.75$ bits / symbol

Huffman Code Example

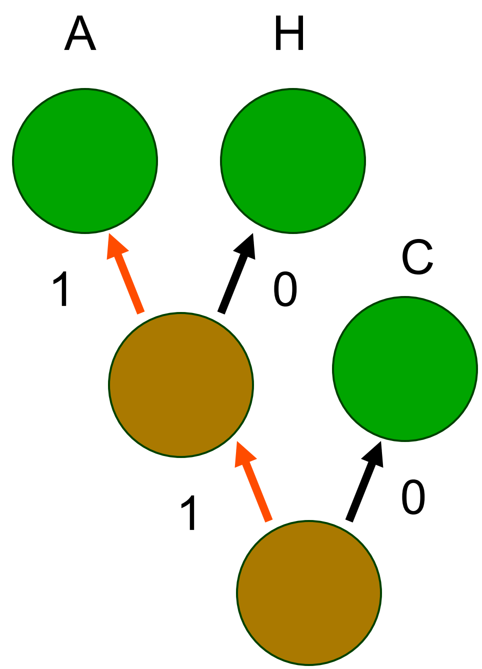
Symbol	Dog	Cat	Bird	Fish
Frequency	1/8	1/4	1/2	1/8
Original Encoding	00	01	10	11
	2 bits	2 bits	2 bits	2 bits
Huffman Encoding	110	10	0	111
	3 bits	2 bits	1 bit	3 bits

How many bits are saved using the above Huffman coding for the sequence Dog Cat Bird Bird Bird?

A. 0 B. 1 C. 2 D. 3 E. 4

Huffman Code Algorithm: Data Structures

- Binary (Huffman) tree
  - Represents Huffman code
  - Edge ⇒ code (0 or 1)
  - Leaf ⇒ symbol
  - Path to leaf ⇒ encoding
  - Example
    - A = "11", H = "10", C = "0"
    - Good when ???
      - A, H less frequent than C in messages
- Want to efficiently build a binary tree



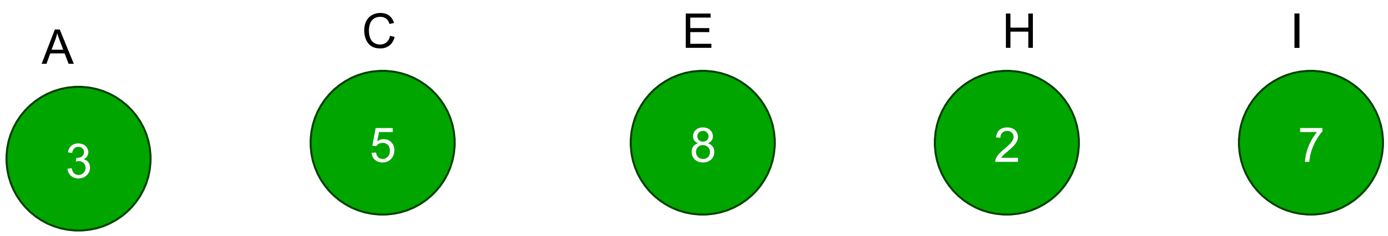
Huffman Code Algorithm Overview

- Order the symbols with least frequent first (will explain)
- Build a tree piece by piece…
- Encoding
  - Calculate frequency of symbols in the message, language
  - JUST COUNT AND DIVIDE BY TOTAL NUMBER OF SYMBOLS
  - Create binary tree representing “best” encoding
  - Use binary tree to encode compressed file
    - For each symbol, output path from root to leaf
    - Size of encoding = length of path
  - Save binary tree

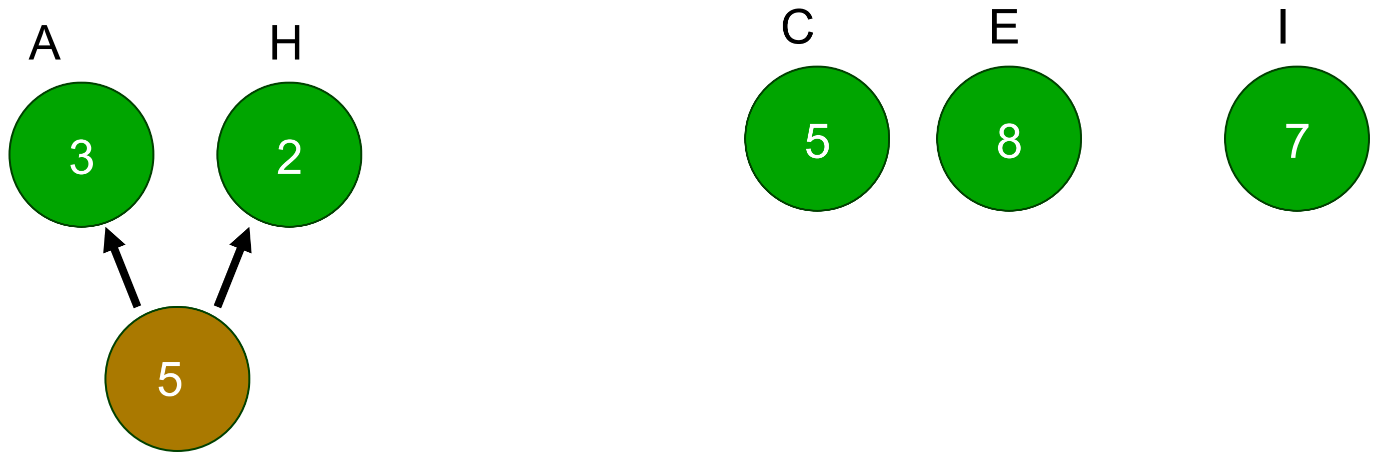
Huffman Code – Creating Tree

- Algorithm (Recipe)
  - Place each symbol in leaf
    - Weight of leaf = symbol frequency
  - Select two trees L and R (initially leafs)
    - Such that L, R have lowest frequencies among all tree
    - Which L, R have the lowest number of occurrences in the message?
  - Create new (internal) node
    - Left child $\Rightarrow$ L
    - Right child $\Rightarrow$ R
    - New frequency $\Rightarrow$ frequency(L) + frequency(R)
  - Repeat until all nodes merged into one tree

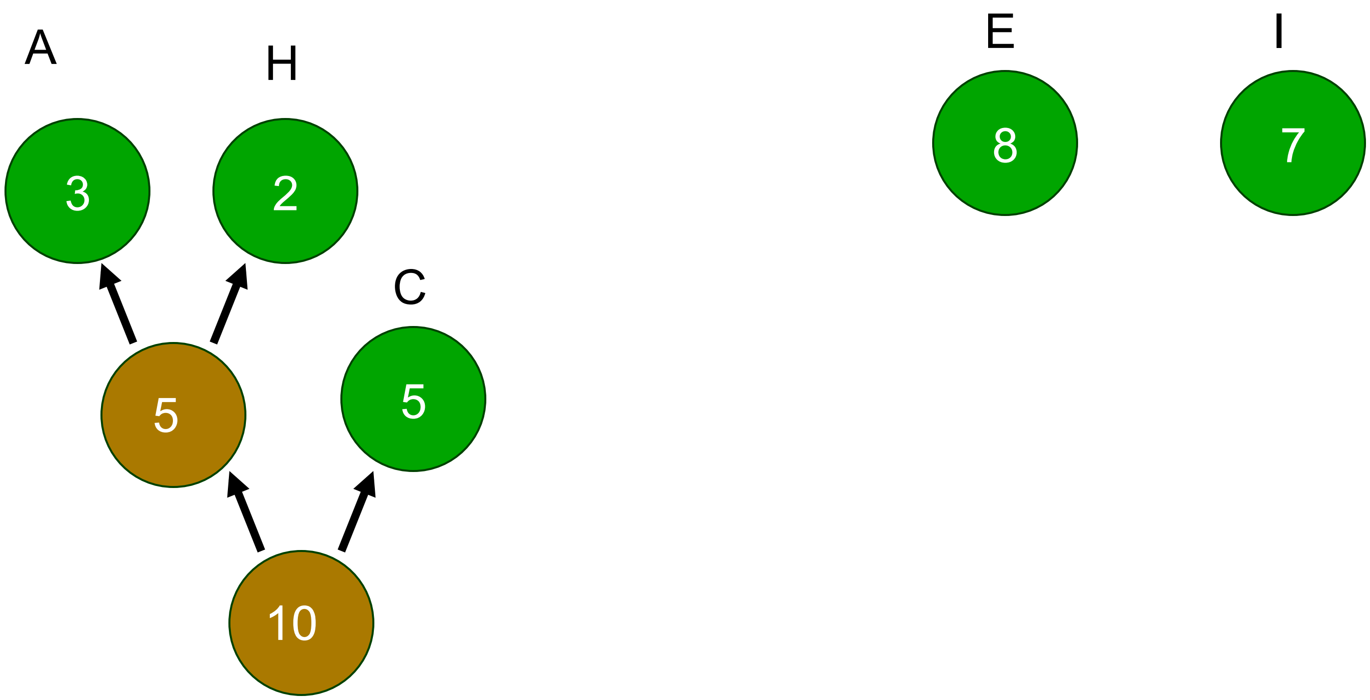
Huffman Tree Construction 1



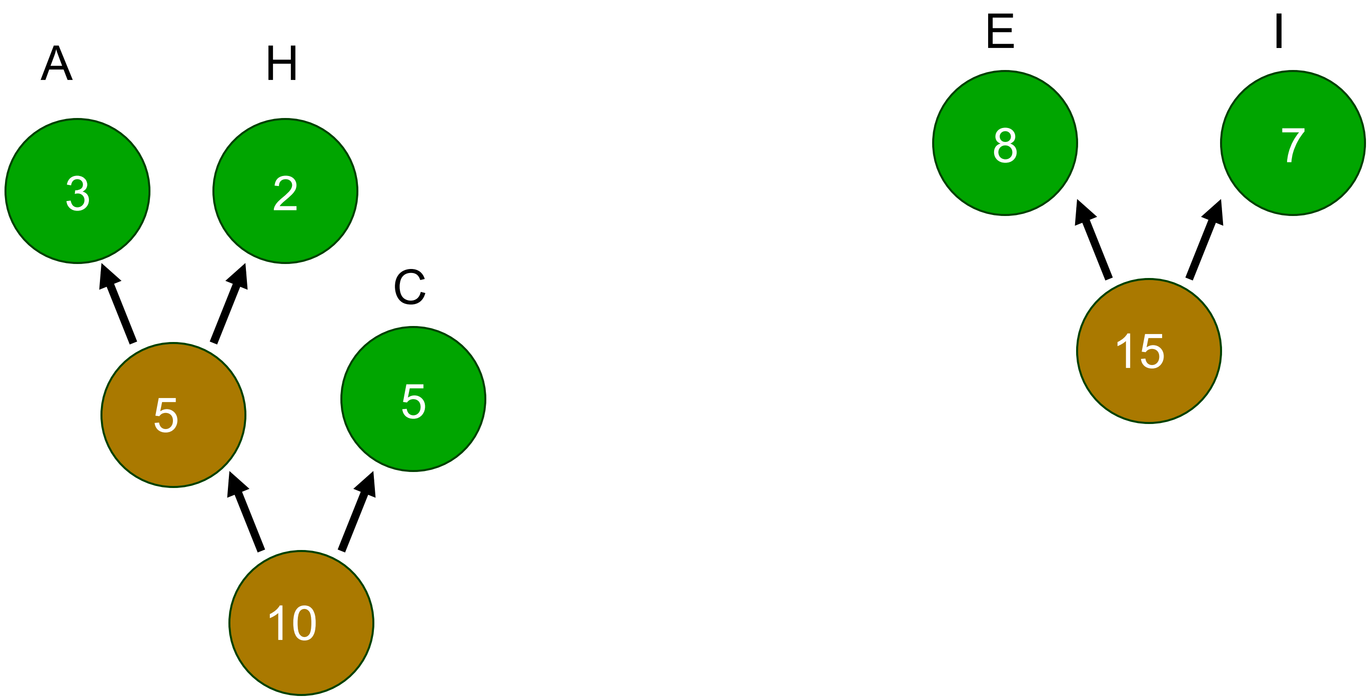
Huffman Tree Step 2: can first re-order by frequency



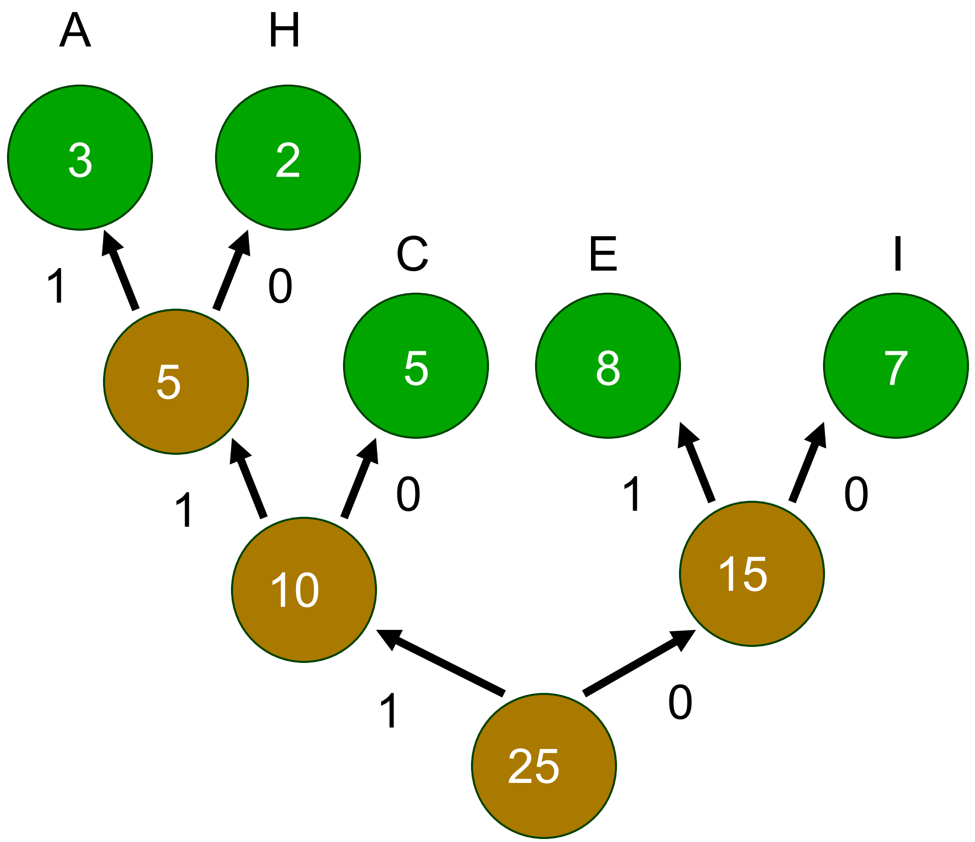
Huffman Tree Construction 3



Huffman Tree Construction 4



Huffman Tree Construction 5



E	=	01
I	=	00
C	=	10
A	=	111
H	=	110

Huffman Coding Example

- Huffman code

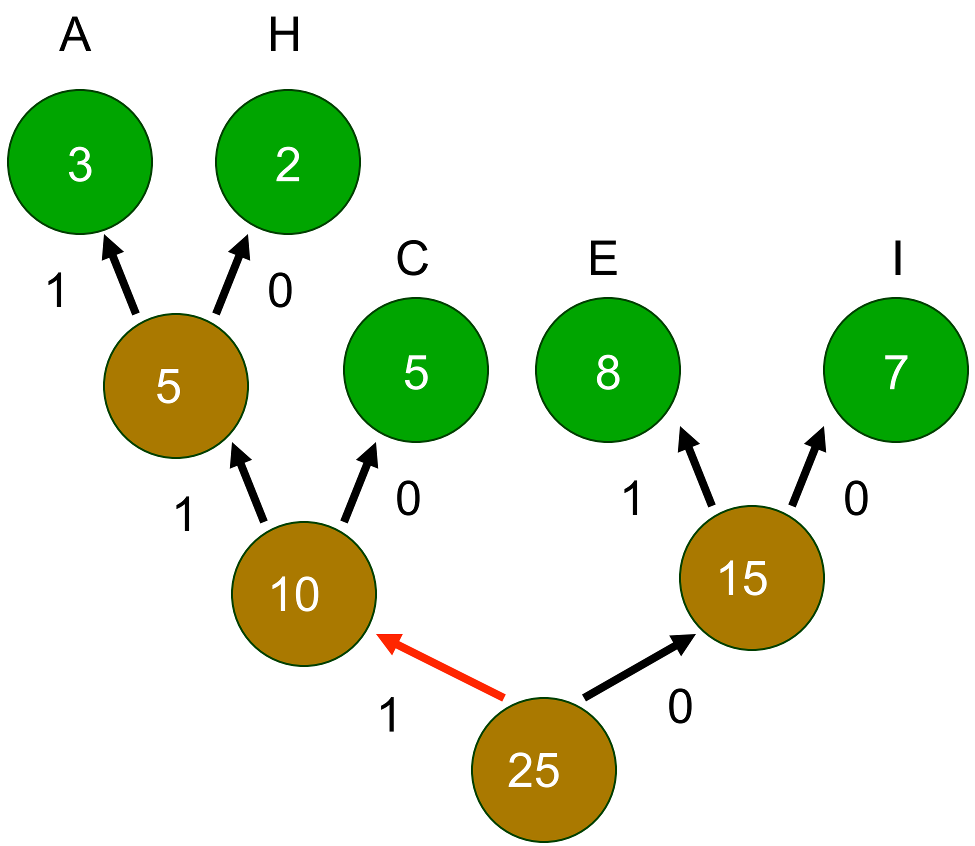
E	=	01
I	=	00
C	=	10
A	=	111
H	=	110

- Input
  - ACE
- Output
  - (111)(10)(01) = 1111001

Huffman Code Algorithm Overview

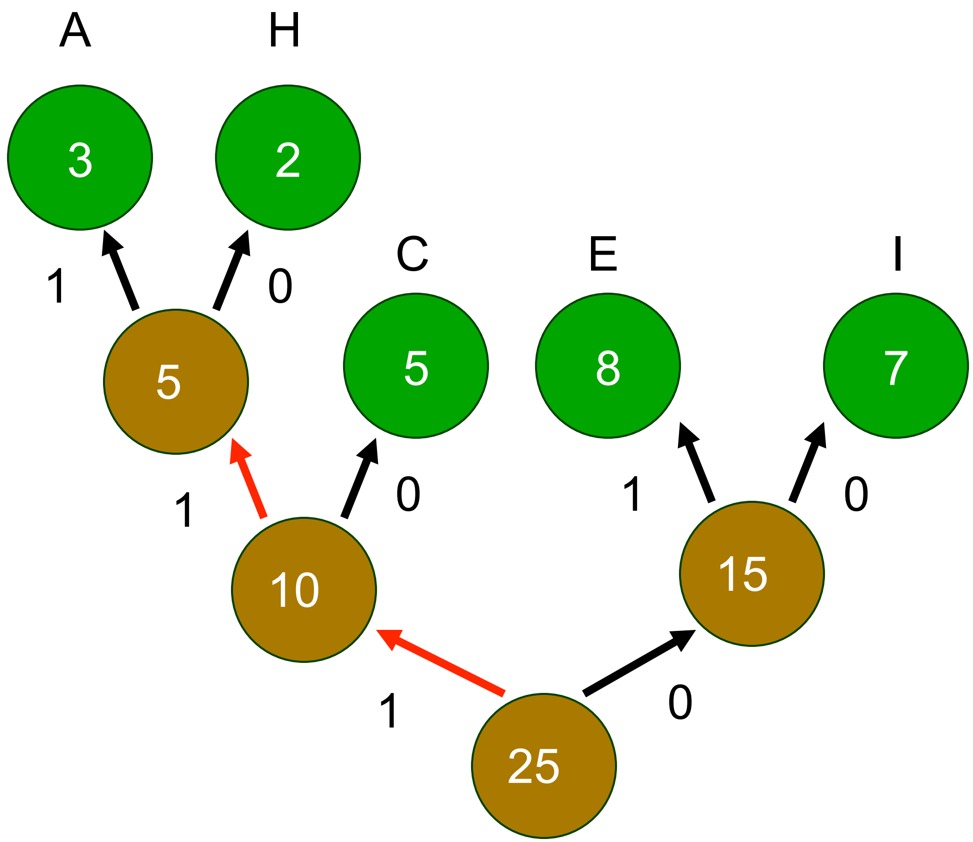
- Decoding
  - Read compressed file & binary tree
  - Use binary tree to decode file
    - Follow path from root to leaf

Huffman Decoding I



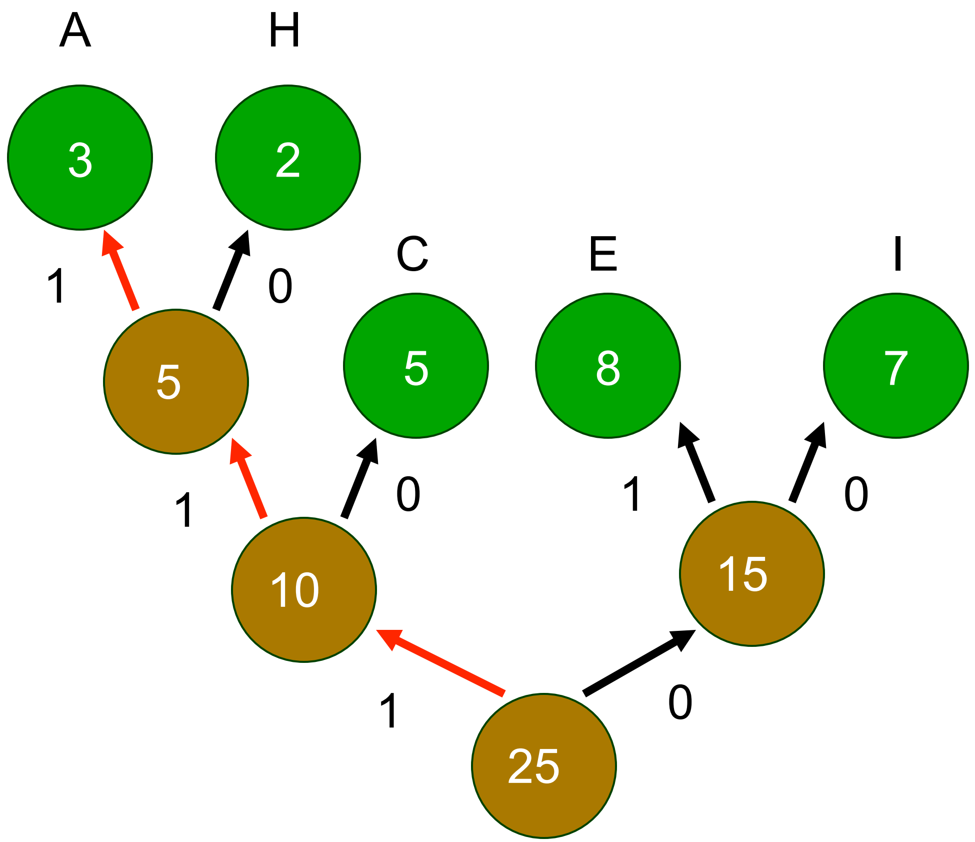
1111001

Huffman Decoding 2



1111001

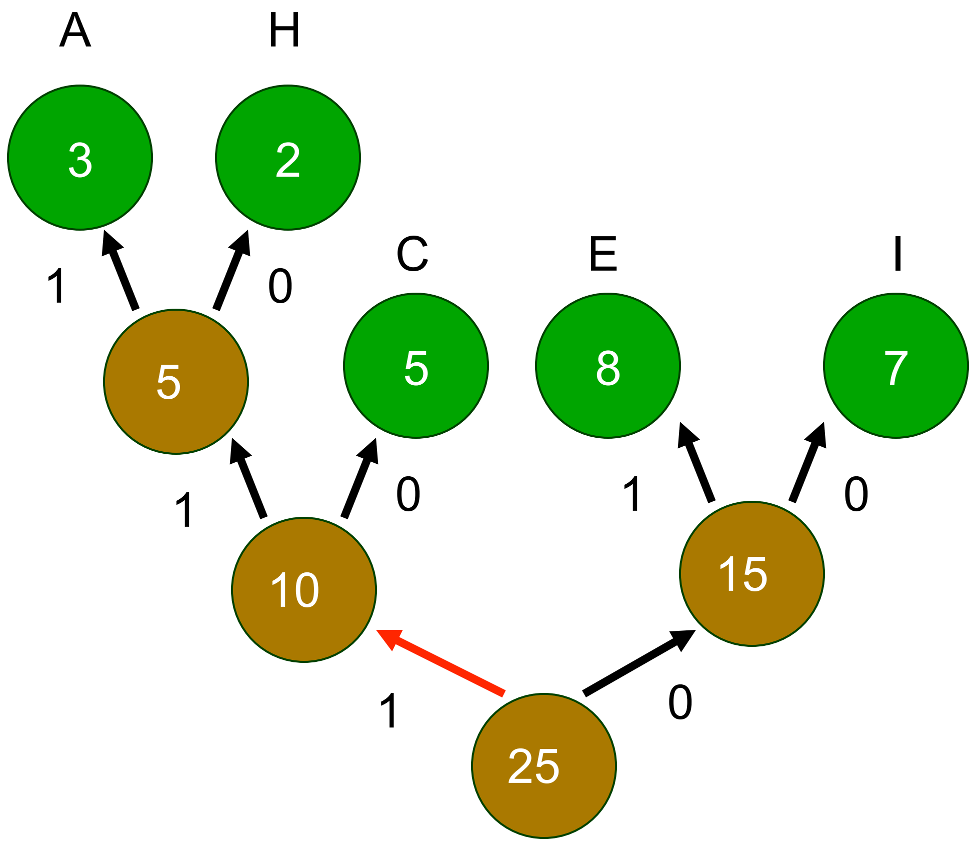
Huffman Decoding 3



1111001

A

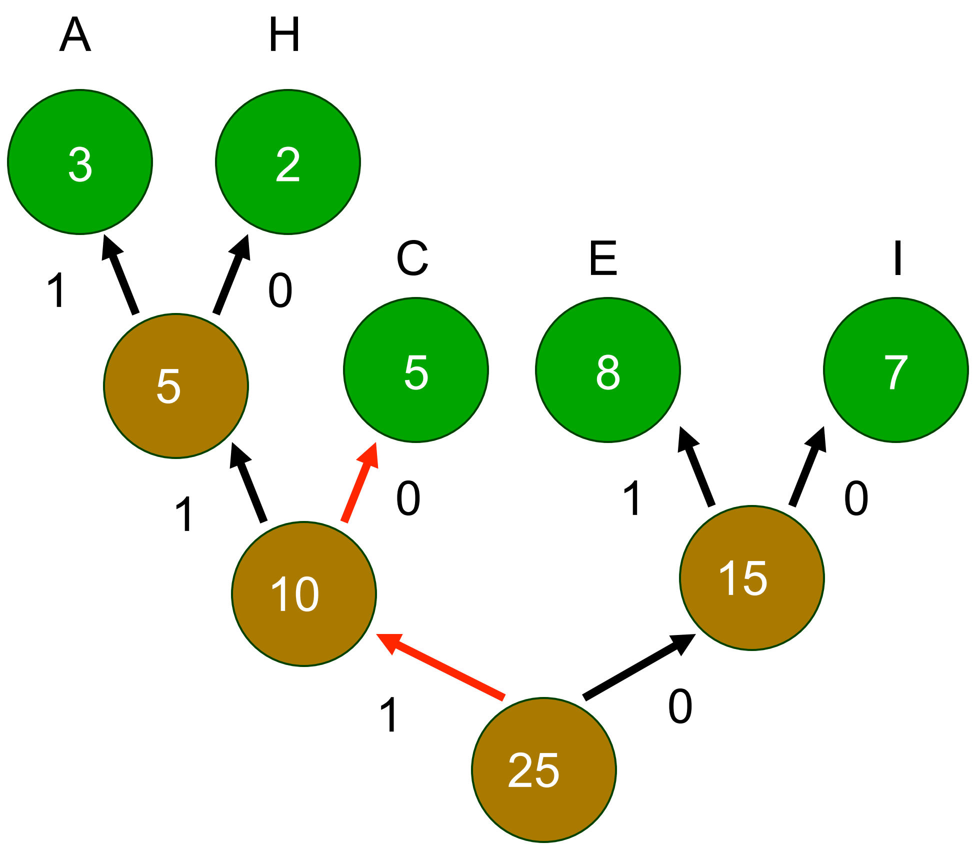
Huffman Decoding 4



1111001

A

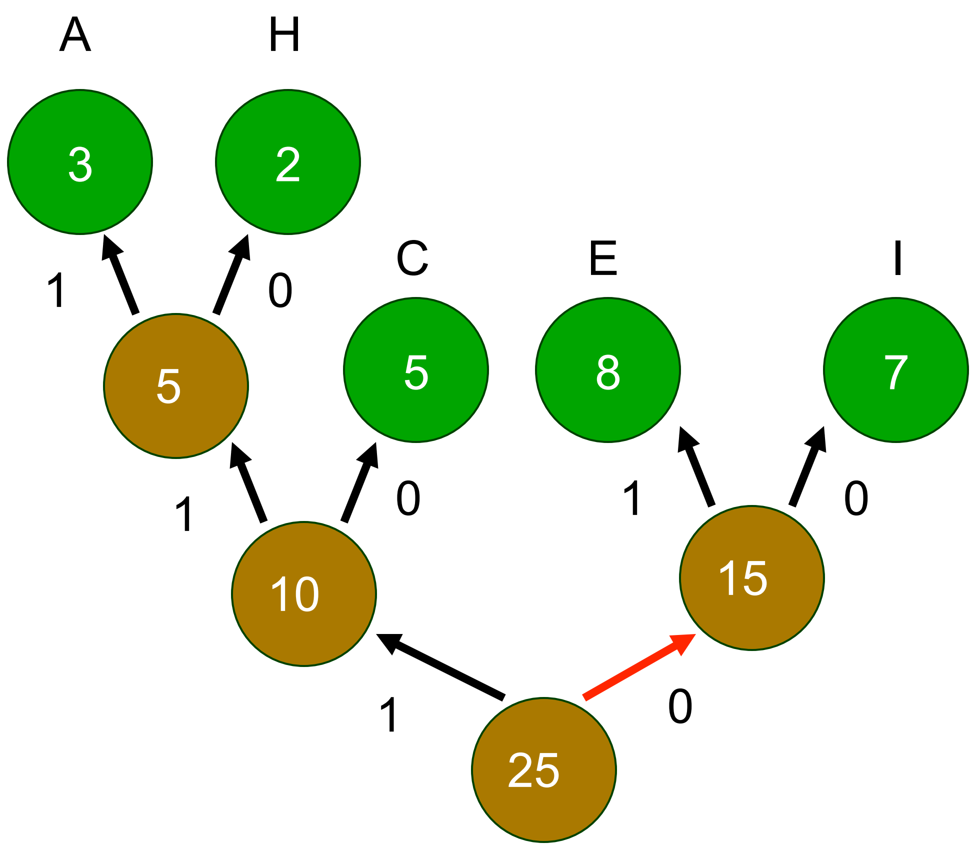
Huffman Decoding 5



1111001

AC

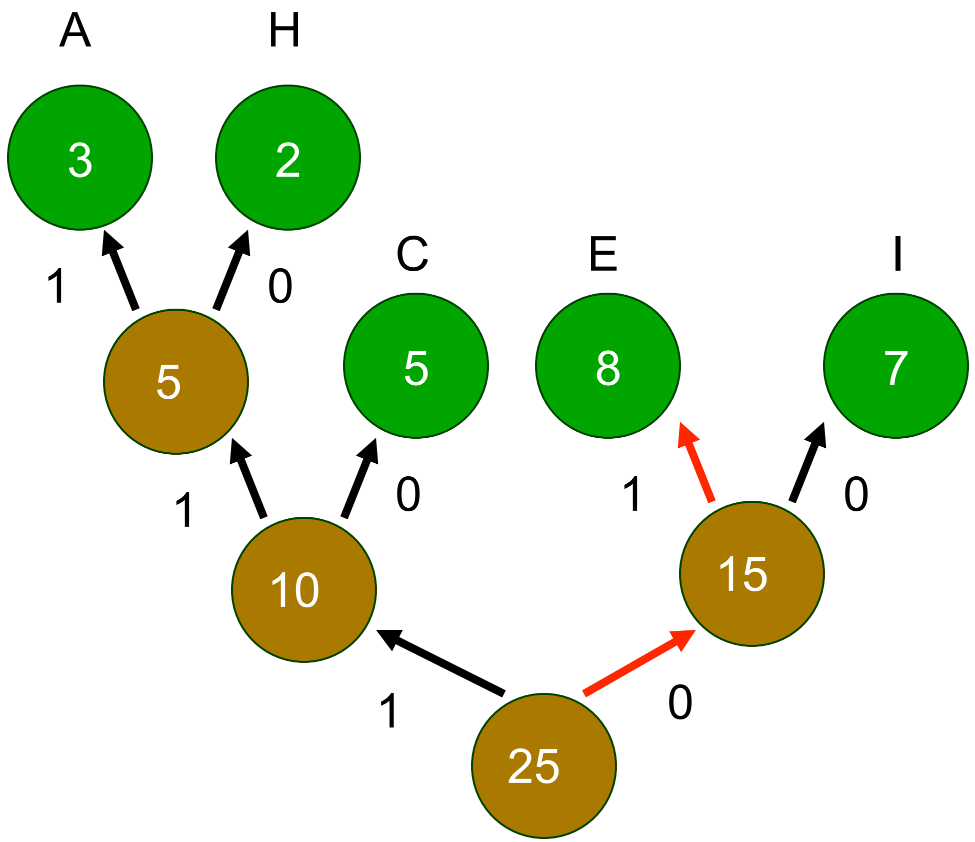
Huffman Decoding 6



1111001

AC

Huffman Decoding 7



1111001

ACE

Huffman Code Properties

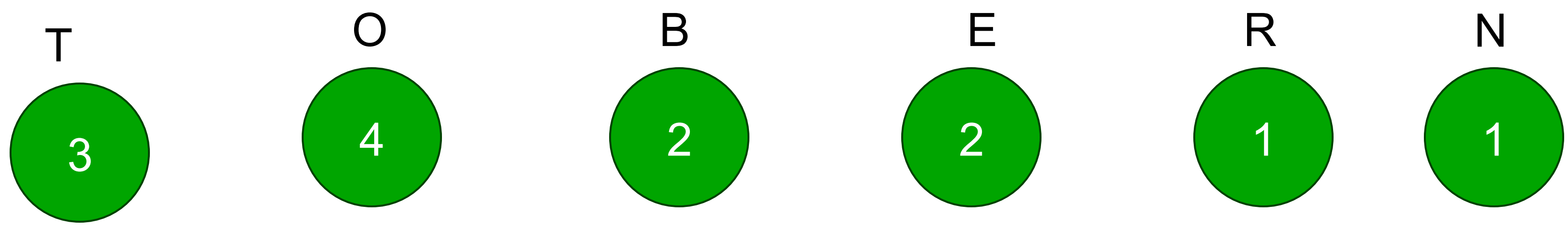
- Prefix code
  - No code is a prefix of another code
  - Example
    - Huffman("dog") ⇒ 01
    - Huffman("cat") ⇒ 011 // not legal prefix code
  - Can stop as soon as complete code found
  - No need for end-of-code marker
- Nondeterministic
  - Multiple Huffman coding possible for same input
  - If more than two trees with same minimal weight

Huffman Code Properties

- Greedy algorithm
  - Chooses best local solution at each step
  - Combines 2 trees with lowest frequency
- Still yields overall best solution
  - Optimal prefix code
  - Based on statistical frequency
- Better compression possible (depends on data)
  - Using other approaches (e.g., pattern dictionary)

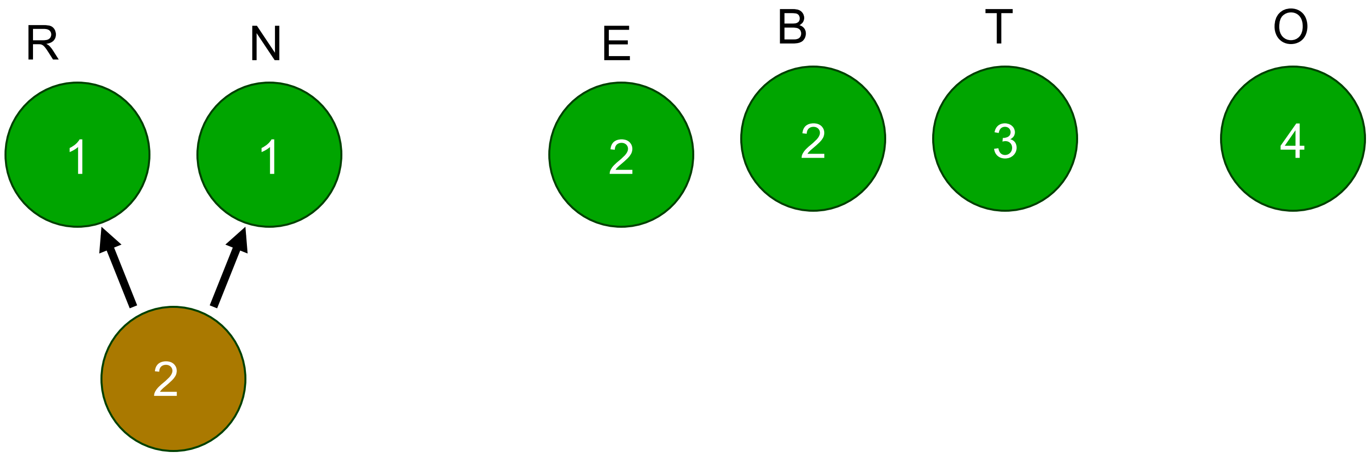
Huffman Coding. Another example.

Huffman Tree Example 2. Step 1

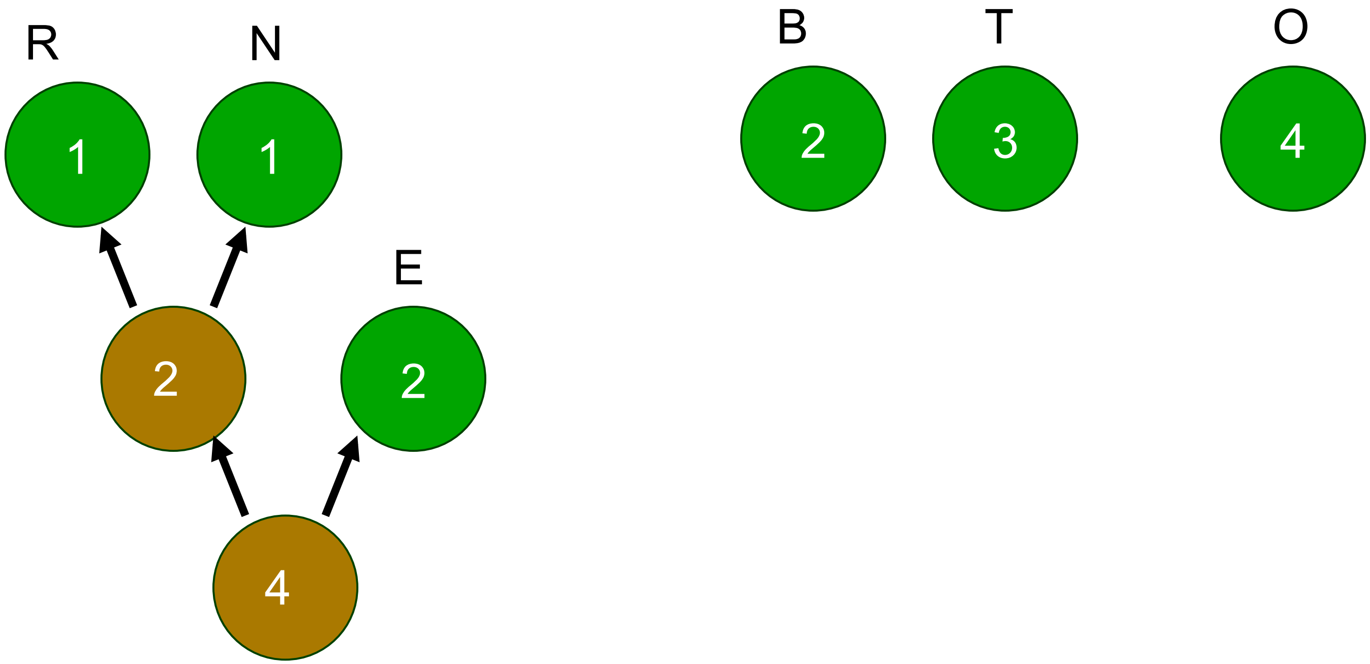


- "TO BE OR NOT TO BE"
  - T = 3
  - O = 4
  - B = 2
  - E = 2
  - R = 1
  - N = 1

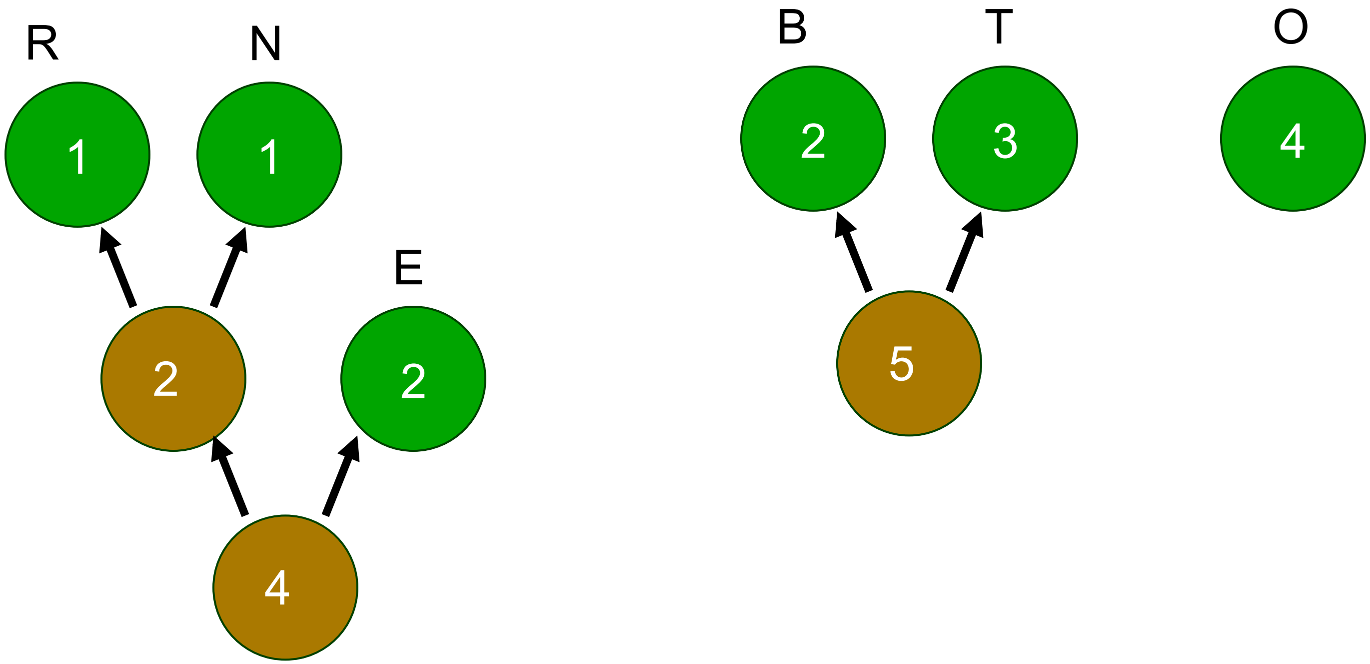
Huffman Tree: TO BE OR NOT TO BE



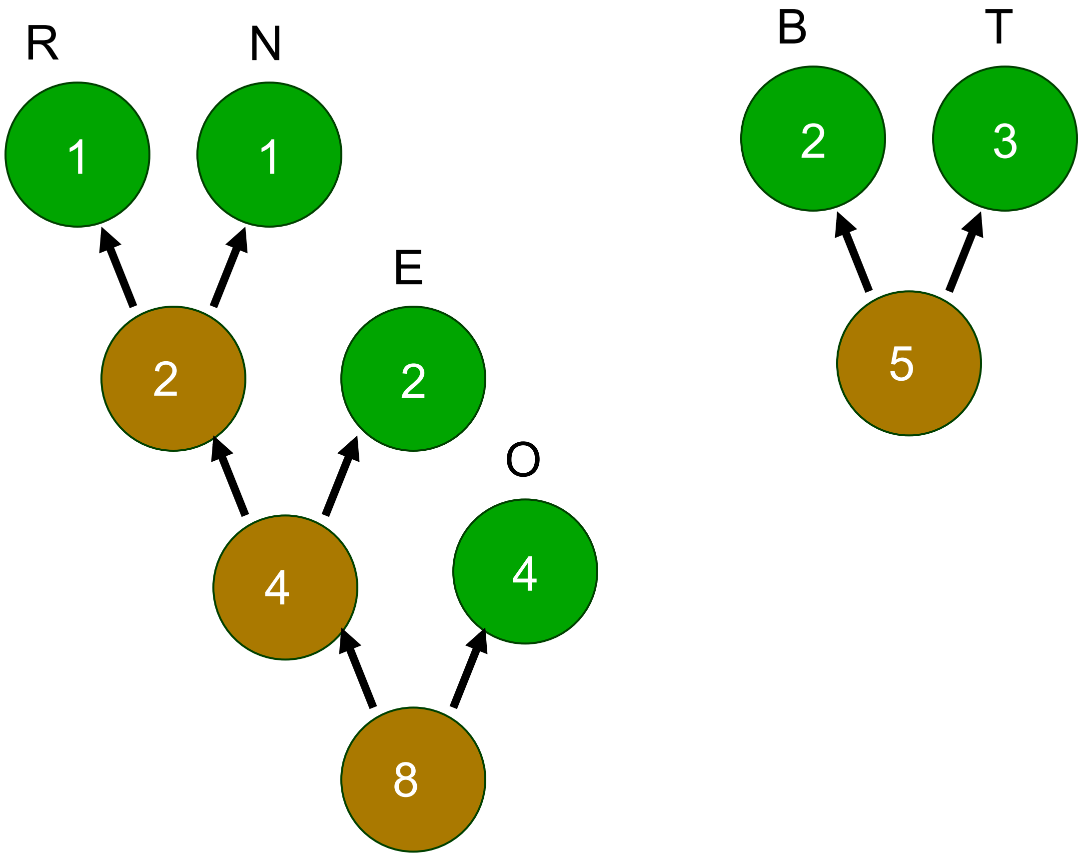
Huffman Tree: TO BE OR NOT TO BE



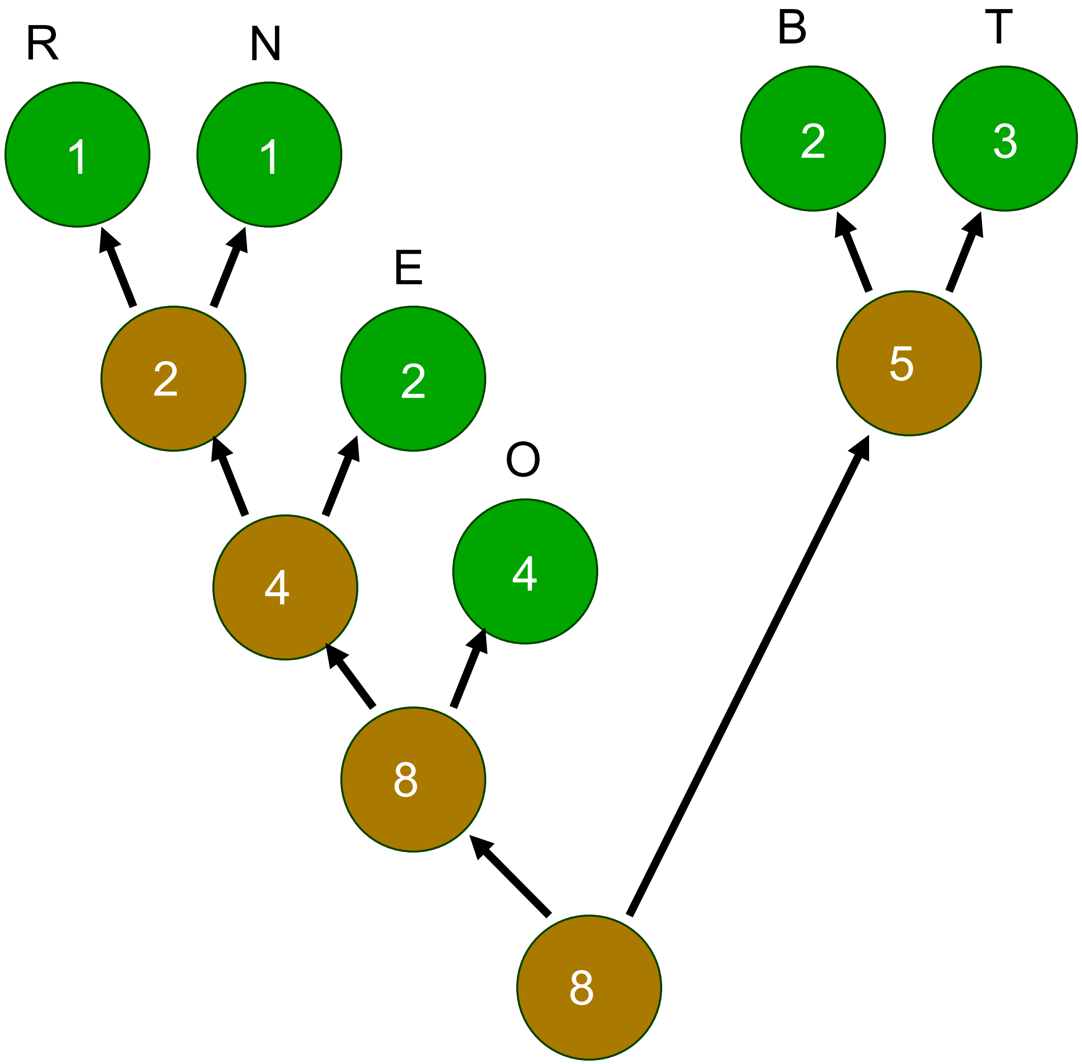
Huffman Tree: TO BE OR NOT TO BE



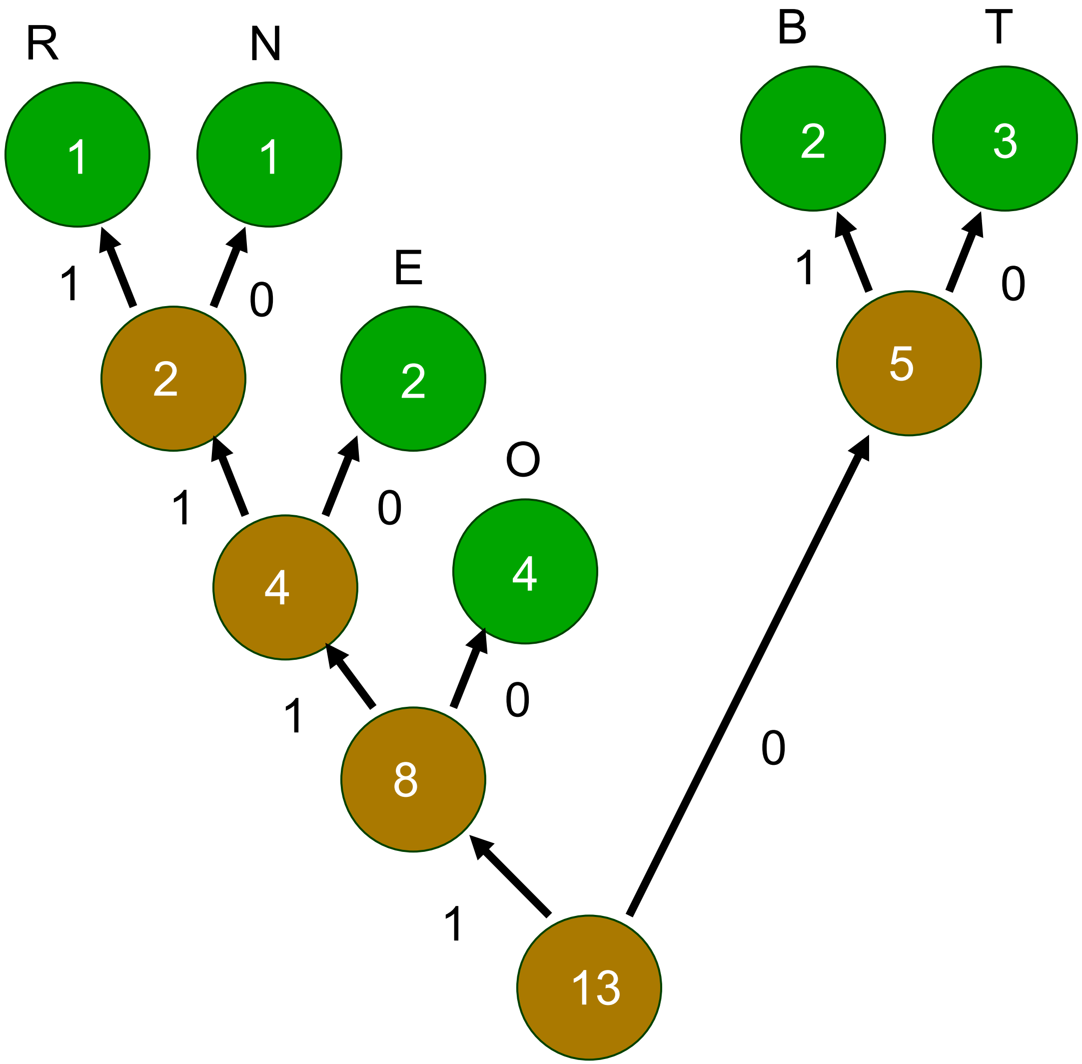
Huffman Tree: TO BE OR NOT TO BE



Huffman Tree: TO BE OR NOT TO BE

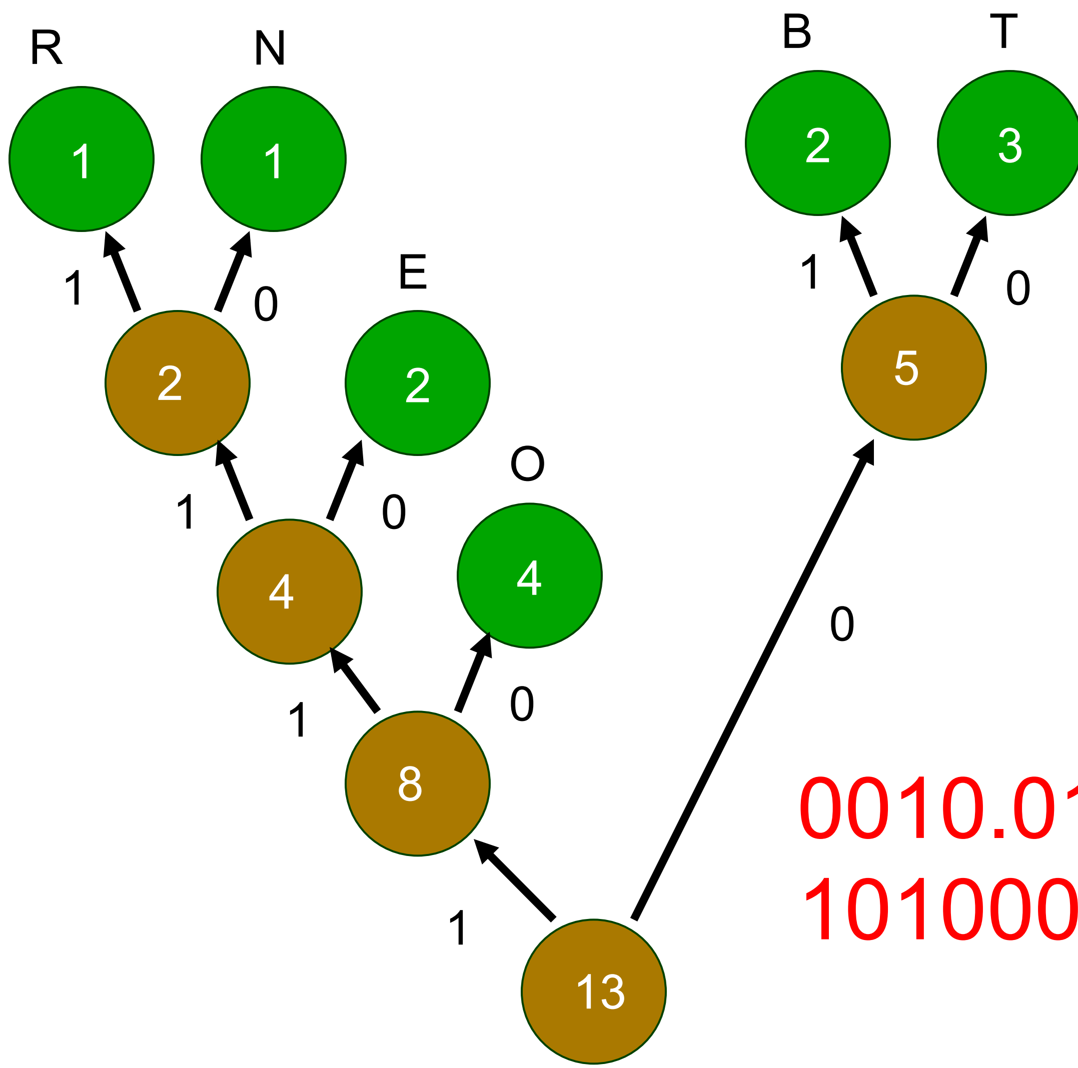


Huffman Tree: TO BE OR NOT TO BE



N	=	1110
R	=	1111
E	=	110
B	=	01
O	=	10
T	=	00

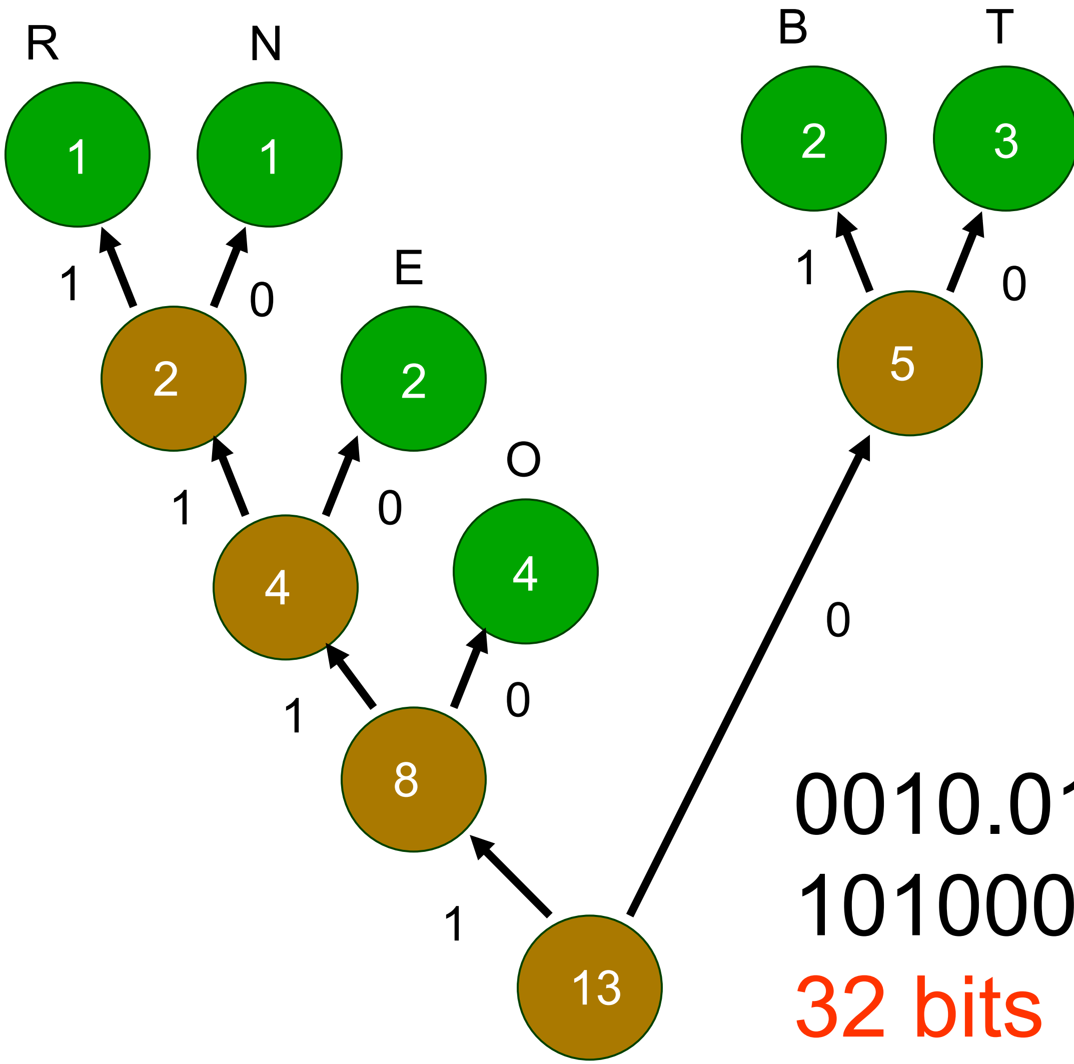
Huffman Tree: TO BE OR NOT TO BE



N = 1110
R = 1111
E = 110
B = 01
O = 10
T = 00

0010.01110.101111.11
101000.0010.01110

Huffman Tree: TO BE OR NOT TO BE



N = 1110
R = 1111
E = 110
B = 01
O = 10
T = 00

0010.01110.101111.11
101000.0010.01110
32 bits

TO BE OR NOT TO BE

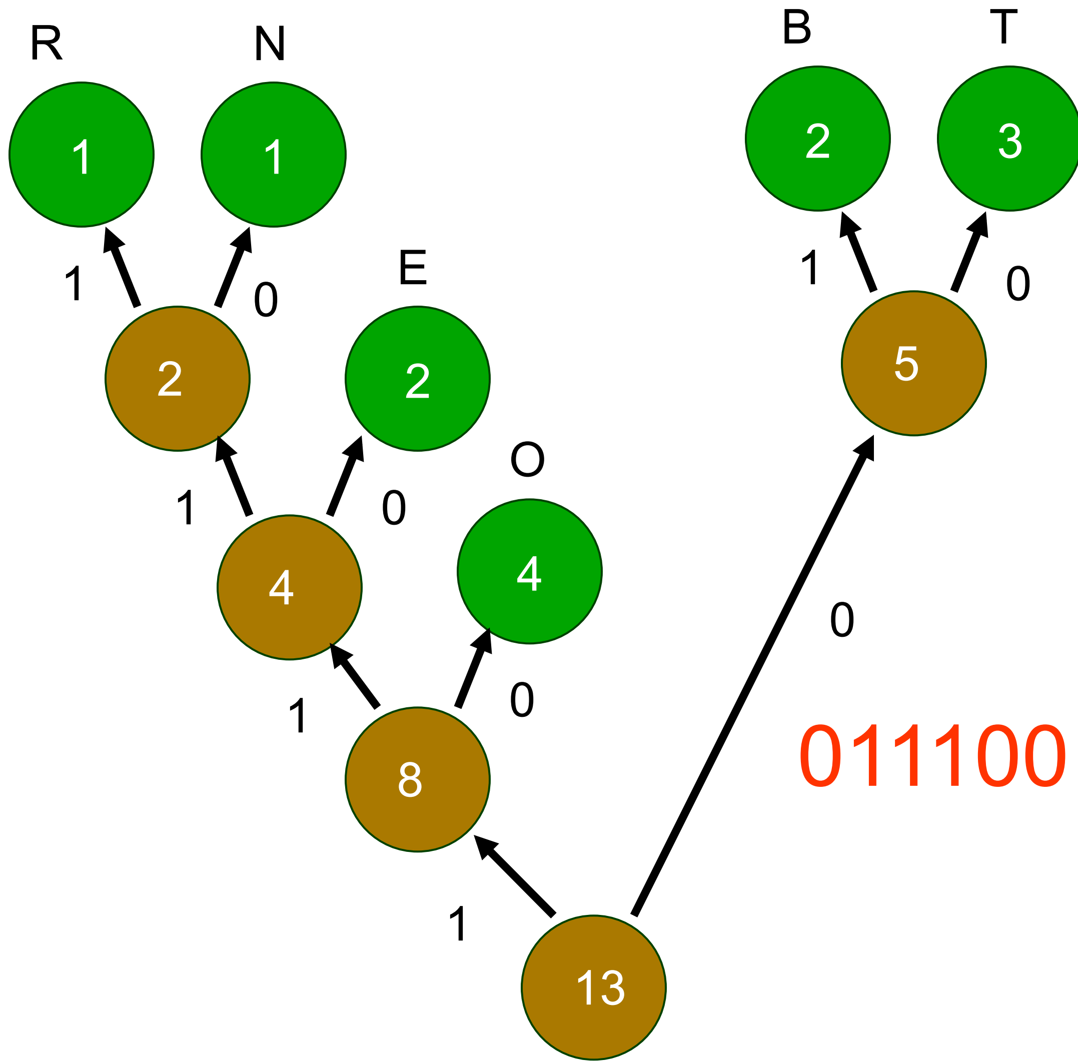
How many bits would it take to store this message if every letter was represented with the same number of bits? You should first figure out how many bits it takes to represent 6 different values/letters.

A. 26
B. 32
C. 39
D. 48
E. 52

N = 1110
R = 1111
E = 110
B = 01
O = 10
T = 00

0010.01110.101111.11
101000.0010.01110
32 bits

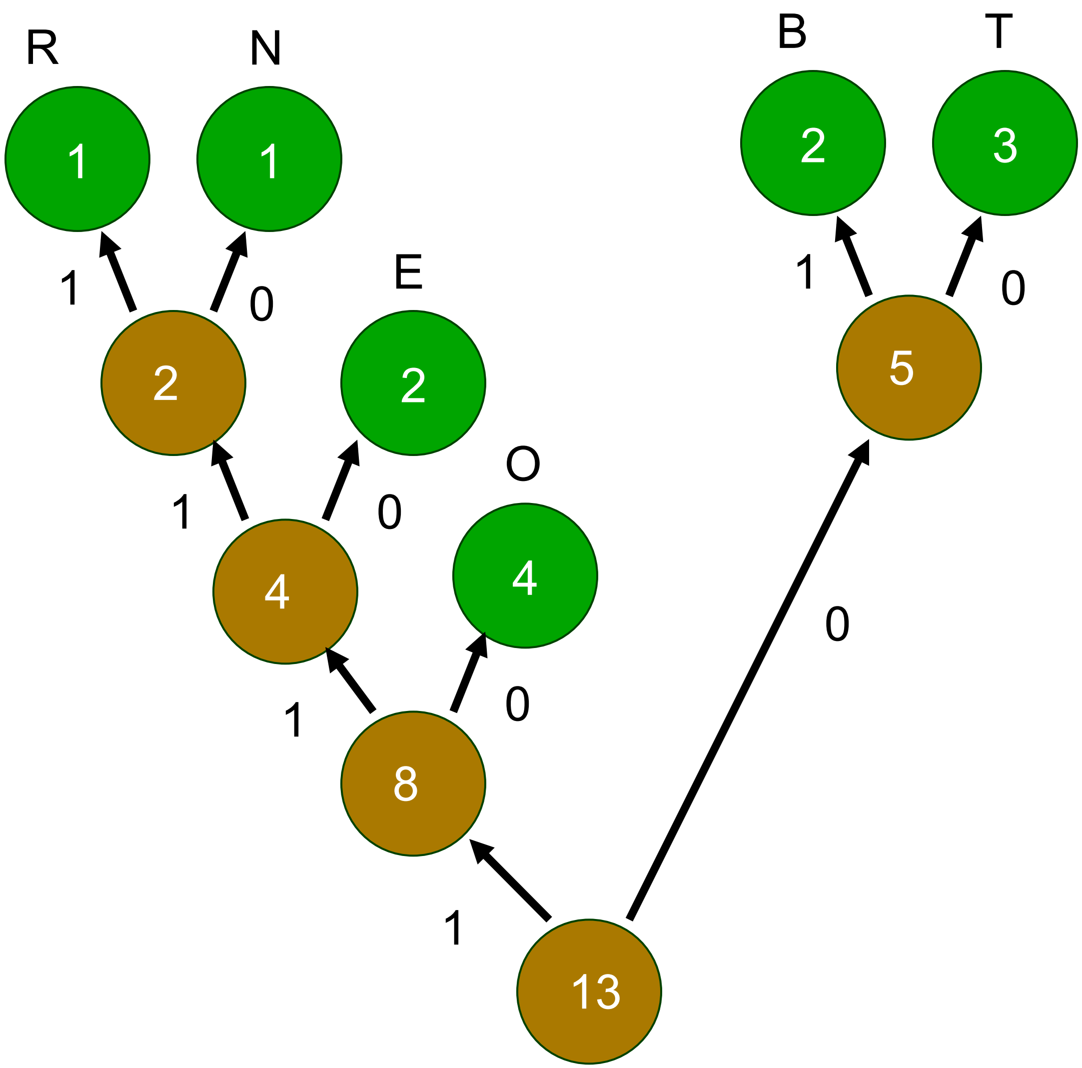
DECODING: 2nd example "BEBE"



N = 1110
R = 1111
E = 110
B = 01
O = 10
T = 00

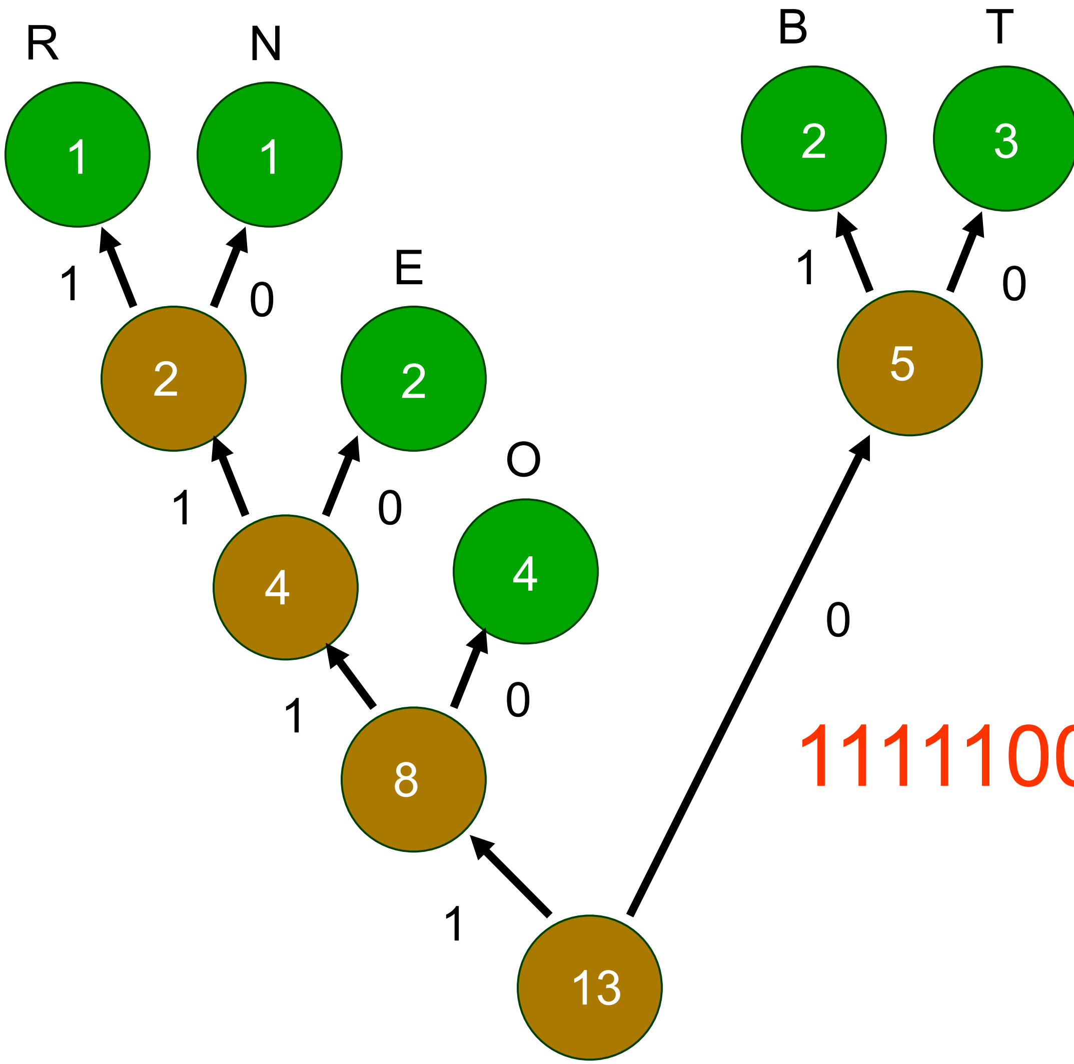
0111001110 = ?

No code is prefix of another



N = 1110
R = 1111
E = 110
B = 01
O = 10
T = 00

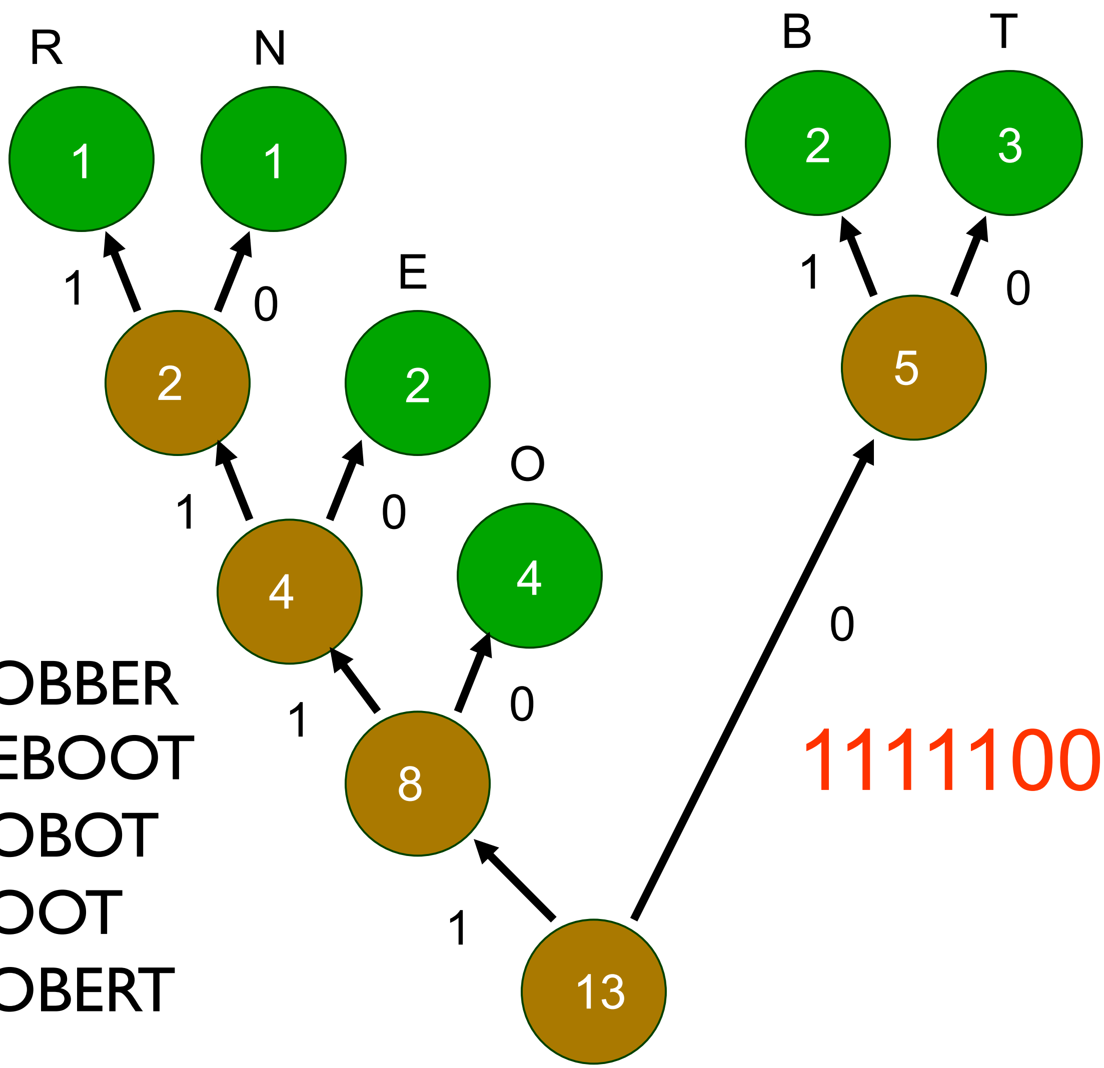
DECODING: Your turn



N = 1110
R = 1111
E = 110
B = 01
O = 10
T = 00

111110011000 = ?

DECODING: Your turn



N = 1110
R = 1111
E = 110
B = 01
O = 10
T = 00

111110011000 = ?

A. ROBBER
B. REBOOT
C. ROBOT
D. ROOT
E. ROBERT